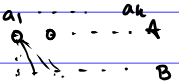
1) G simple d-regular is bipartite iff it has a decomposition into copies of $K_{1,d}$

If G is bipartite



$A = \{a_1, \dots, a_k\}$

It's clear $\{a_i, N(a_i)\} \cong K_{1,d}$ These are edge disjoint

$\uparrow$ Neighborhood

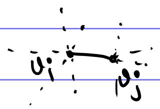
Every edge is adjacent to some a_i, so it's in some $K_{1,d}$. Thus G is decomposed into

$$G = \bigcup_{i=1}^{k} \Big(\underbrace{\{a_i, N(a_i)\}}_{\text{vertices}}, \underbrace{\{a_i y : y \in N(a_i)\}}_{\text{edges}} \Big)$$

Suppose G has a decomposition into k copies of $K_{1,d}$

Let $v_1 \dots v_k$ be the centers of these $K_{1,d}$.

Claim: $v_i v_j$ cannot be an edge



If it's an edge say it's in $K_{1,d}(v_i)$

Then it cannot also be in $K_{1,d}(v_j)$ since it's a decomposition. Then $d(v_j) \geq |E(K_{1,d}(v_j))| + |\{v_i v_j\}| = d+1$

This is a contradiction to $d(v_j) = d$.

Claim: If $x \in V(K_{1,d}(v_i)) \setminus v_i$ $y \in V(K_{1,d}(v_j)) \setminus v_j$ then xy is not an edge.

Pf: If it is then either x or y must be in some $K_{1,d}(v_k)$ for $k \neq i, j$. But this means $v_i v_k$ or $v_j v_k$ must be an edge, which is not allowed by the previous claim.

So $\left(\{v_i\}_{i=1}^{k} , \{V(K_{1,d}(v_i)) \setminus v_i\}_{i=1}^{k} \right)$ are a bipartition